

NOTATION

L, d , vortex tube length and diameter; d_d , diaphragm diameter; L_d , distance between the tangential inlet channels and the location of the pressure transducer; F_{in} , area of the tangential inlet channels; ΔP , pressure drop in the vortex tube; $\Delta P'_{h.f.}$, $\Delta P'_{l.f.}$, $\Delta P'_{\Sigma}$, amplitudes of the low- and high-frequency pressure fluctuations and the total pressure fluctuation level; $f_{l.f.}$, $f_{h.f.}$, low- and high-frequency pressure fluctuation frequencies; μ , relative mass flow rate of the cooled air; and ΔT_x , ΔT_g , air cooling and heating effects in the vortex tube.

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TURBULENT FLUID FLOW IN A CIRCULAR PIPE WITH UNIFORM BLOWING THROUGH POROUS WALLS

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The average and fluctuating incompressible fluid flow characteristics in a circular pipe with blowing are computed on the basis of a three-parameter model of turbulence.

Investigations of flow in channels with permeable walls are of interest for the analysis of heat- and mass-transfer processes in heat pipes when using blowing in the interest of heat shielding and in many other applications. Computations of the turbulent flow in pipes with blowing have been performed in [1, 2] on the basis of mixing-path length models, in [3] for the transition flow mode, and in [4] for the hydrodynamically stabilized stream by using additional equations for the fluctuating motion. Flow development along the pipe length is investigated in this paper for relatively high Reynolds number of the main stream at the input for conditions that are almost realized in experiments [5].

Solutions for the equations of average and fluctuating motion have been obtained in the boundary-layer theory approximation valid for $m \ll 1$. The fluctuating motion is described by a three-parameter model of turbulence, consisting of the equations of fluctuating energy balance, turbulent tangential stresses, and turbulent energy dissipation, described in a form close to the models proposed earlier in [6, 7]. The system of equations used in the computations for axisymmetric stationary incompressible fluid flow in a circular pipe has the form

$$\frac{\partial(ru_x)}{\partial x} + \frac{\partial(ru_r)}{\partial r} = 0, \quad (1)$$

$$u_x \frac{\partial u_x}{\partial x} + u_r \frac{\partial u_x}{\partial r} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(\nu \frac{\partial u_x}{\partial r} - \sigma \right) \right], \quad (2)$$

$$u_x \frac{\partial E}{\partial x} + u_r \frac{\partial E}{\partial r} = -\sigma \frac{\partial u_x}{\partial r} - \frac{cE^{3/2}}{L} - \frac{c_{1E}\nu E}{L^2} + \frac{1}{r} \frac{\partial}{\partial r} \left[r(\nu + \alpha_E E^{1/2} L) \frac{\partial E}{\partial r} \right], \quad (3)$$

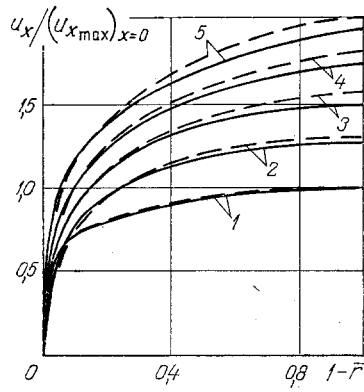


Fig. 1

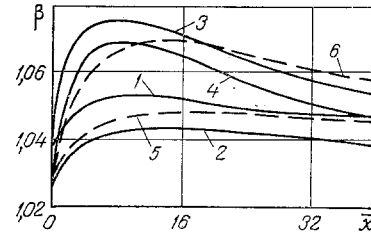


Fig. 2

Fig. 1. Axial velocity distribution over the pipe sections: 1) $\bar{x} = 0$; 2) 8; 3) 16; 4) 24; 5) 32.

Fig. 2. Change in the momentum flux coefficient over the pipe length: 1) $Re_0 = 3 \cdot 10^4$, $m_0 = 0.005$; 2) $8 \cdot 10^4$ and 0.005 ; 3) $3 \cdot 10^4$ and 0.012 ; 4) $8 \cdot 10^4$ and 0.012 ; 5) $m_0 = 0.00466$; 6) 0.0121 .

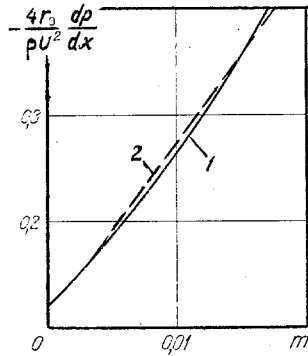


Fig. 3

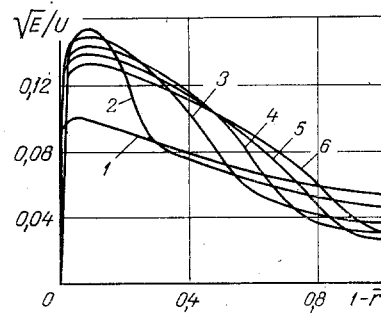


Fig. 4

Fig. 3. Dependence of the pressure gradient on the blowing parameter: 1) computation; 2) experiment [5].

Fig. 4. Turbulent energy distribution over the pipe sections: 1) $\bar{x} = 0$; 2) 4; 3) 12; 4) 20; 5) 28; 6) 36.

$$u_x \frac{\partial \sigma}{\partial x} + u_r \frac{\partial \sigma}{\partial r} = -k_1 [1 - \exp(-\gamma Re_E)] E \frac{\partial u_x}{\partial r} - \frac{kE^{1/2}\sigma}{L} - \frac{c_{1\sigma} v \sigma}{L^2} + \frac{1}{r} \frac{\partial}{\partial r} \left[r(v + \alpha_\sigma E^{1/2} L) \frac{\partial \sigma}{\partial r} \right] - \frac{(v + \alpha_\sigma E^{1/2} L)}{r^2} \sigma, \quad (4)$$

$$u_x \frac{\partial F}{\partial x} + u_r \frac{\partial F}{\partial r} = -\frac{\alpha F}{E} \frac{\partial u_x}{\partial r} - \frac{2cE^{1/2}F}{L} - \frac{c_{1F} v F}{L^2} + \frac{1}{r} \frac{\partial}{\partial r} \left[r(v + \alpha_F E^{1/2} L) \frac{\partial F}{\partial r} \right]. \quad (5)$$

The factor $1 - \exp(-\gamma Re_E)$ is introduced to diminish the contribution of generation to the balance of Reynolds stresses in the viscous sublayer region. The scale of turbulence is normalized in such a way that in the absence of blowing the value of L would agree with the Prandtl mixing path length in the near-wall domain, i.e., $L = 0.4y$ as $y \rightarrow 0$. The constants in (3)-(5) have the following values: $\alpha_E = \alpha_\sigma = \alpha_F = 0.2$; $c = 0.13$; $c_{1E} = 0.32$; $k_1 = 0.2$; $\gamma = 0.06$; $k = 0.35$; $c_{1\sigma} = 1.92$; $a = 1.7$; $c_{1F} = 0.32$.

The boundary conditions at the wall and at the pipe axis have the following form for (1)-(5):

$$r = r_0, u_x = E = \sigma = F = 0, u_r = V;$$

$$r = 0, \frac{\partial u_x}{\partial r} = \frac{\partial E}{\partial r} = \sigma = \frac{\partial F}{\partial r} = 0.$$

The boundary conditions at the inlet to the pipe ($x = 0$) are determined from the solution of the equation for $V = 0$, i.e., it is assumed that the porous pipe is a continuation of the channel with impermeable walls. The factorization method with iteration is used for the computation; the pressure gradient in (2) is eliminated here by using the splitting method [8].

The distributions obtained for u_x , E and σ for $V = 0$ are in good agreement with the Laufer [9] experimental results. The distribution of the scale of turbulence over the tube section practically agrees with the Prandtl–Nikuradze dependence [10]. The discrepancy between values of the friction drag coefficient and the Filonenko formula [11] does not exceed 2% in the Range of Reynolds numbers $10^4 \leq Re \leq 4 \cdot 10^5$.

Results of computations of the axial velocity normalized to its maximal value at the inlet are compared with experimental distributions [5] (dashes) in Fig. 1 for $Re_0 = 3 \cdot 10^4$ and $m_0 = 0.012$. The change in the momentum flux coefficient along the pipe length, which characterizes the deformation of the velocity profile, is shown in Fig. 2. The course of the change in β with respect to \bar{x} agrees qualitatively with the experimental data [5] (dashes). The growth of β on the initial section is related to the diminution in the population of the axial velocity profile during adjustment of u_x under the effect of blowing from the input distribution corresponding to the flow for $V = 0$. Diminution of β after passage through the maximum is explained by the circumstance that the flow becomes quasistabilized and the velocity distribution in each section corresponds to the local blowing parameter m , whose value drops along the pipe length. The dependence on the parameter m obtained in the computations for the pressure gradient (Fig. 3) and the friction drag coefficient are in good agreement with experimental results [5].

The turbulent energy distribution normalized relative to the local value of the mean velocity is represented in Fig. 4 for $Re_0 = 3 \cdot 10^4$ and $m_0 = 0.02$ over different pipe sections. As in boundary-layer flows on permeable surfaces [12–15], under the effect of blowing the fluctuation maximum is shifted somewhat from the wall to the stream, and its value increases. In the viscous sublayer domain the degree of turbulence diminishes since the fluctuations are forced back from the wall by the blown fluid. The turbulence level is reduced in the near-axis zone, which is observed in experiments [15] in a rectangular channel with one-sided blowing. The diminution in the degree of turbulence at the center of the pipe is related to stream acceleration because of growth of the mass flow rate along the pipe length and is manifested by means of the convective fluctuation transport mechanism in the axial direction. The influence of blowing on the Reynolds stress distribution over the pipe section and length is the same, with the exception of on the axis, as in turbulent energy. The scale of turbulence in the near-wall domain varies negligibly, and diminishes at the core of the stream; an analogous effect of blowing on the mixing path length in the boundary layer is experimentally established in [16].

NOTATION

x, r , longitudinal and radial coordinates; u_x, u_r , longitudinal and radial velocity components; p , pressure; ν , coefficient of kinematic viscosity; E , turbulent energy; $\sigma = \langle u'_x u'_r \rangle$, turbulent tangential stress; $F = E^{3/2}/L$, dissipative function; L , scale of turbulence; r_0 , pipe radius; U_0 , mean velocity at the pipe inlet; $U = U_0 - 2xV/r_0$, local mean velocity; V , blowing velocity ($V < 0$); $m = V/U$, blowing parameter; $Re = 2r_0U/\nu$, main stream Reynolds number; $m_0 = -V/U_0$; $Re_0 = 2r_0U_0/\nu$; $Re_E = E^{1/2}L/\nu$, turbulent Reynolds number; $\bar{x} = x/r_0$, $\bar{r} = r/r_0$, dimensionless coordinates; and $\beta = 2 \int_0^1 u_x^2 \bar{r} d\bar{r} / U^2$, momentum flux coefficient.

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EQUATION FOR THE STRUCTURE FUNCTION OF A TURBULENT
STATIONARY ISOTROPIC VELOCITY FIELD AND ITS SOLUTION
IN THE INERTIAL SCALE INTERVAL

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A closed equation is obtained for the structure function of a turbulent stationary isotropic velocity field and the equation is solved in the inertial scale interval.

1. A closed equation for the structure function of a turbulent isotropic nonstationary velocity field is obtained in [1, 2]. In this paper, we attempt to obtain the stationary form of this equation and solve it in the inertial scale interval.

In order to obtain the stationary form of Eq. (22) in [2] for the structure function $D(r)$, it is necessary to pass in this equation to the limit $t > \infty$ and calculate the integral over the time variable. However, in so doing, it is necessary to take into account the fact that the integrand on the right side of this equation depends on time explicitly and through the function being sought. The temporal dependence of the function $D(r, \tau)$ on the right side of the equation, generally speaking, cannot be neglected, since the integral over τ is calculated from $\tau = 0$ to $t \rightarrow \infty$. It is clear that for τ close to zero, the functions sought depend strongly on time. However, when some certain conditions are satisfied, this dependence can be neglected and the integration over τ can be carried out. We will obtain these conditions.

One of the terms on the right side of Eq. (22) in [2] can be written in the form

$$I_1 \propto \int_0^\infty d\rho \int_0^t \frac{d\tau}{(t-\tau)^3} k_0^1(\rho, n, q) \varphi(\rho, \tau), \quad (1)$$

where k_0^1 is defined by Eq. (28) in [2]; here and in what follows, we will understand the symbol φ to mean the part of the integrand which depends on τ through the functions sought and does not depend on τ explicitly. Changing the variable of integration τ according to the equation $(t-\tau)^{-1} = z$ and using Eq. (28) in [2] for k_0^1 ,

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